

Chiral Plasma Instabilities

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We study the collective modes in relativistic electromagnetic or quark-gluon plasmas with an asymmetry between left- and right-handed chiral fermions, based on the recently formulated kinetic theory with Berry curvature corrections. We find that there exists an unstable mode, signaling the presence of a plasma instability. We argue the fate of this “chiral plasma instability” including the effect of collisions, and briefly discuss its relevance in heavy ion collisions and compact stars.

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Introduction.—Parity-violating effects related to the quantum anomalies play important roles in a wide range of physics from quantum Hall systems to the cosmology. One example in the transport phenomena is the parity-violating current in the presence of a magnetic field and an asymmetry between left and right-handed fermions, characterized by the chiral chemical potential $\mu_5 \equiv \mu_R - \mu_L$. This is called the chiral magnetic effect (CME) [1–4]. Recently hydrodynamics [5] and kinetic theory [6–8] have been appropriately modified to describe quantum anomalies and the CME (see also Refs. [9, 10]); in the kinetic theory, essential corrections are the Berry curvature, the concept diversely applied in condensed matter physics [11]. These developments enable us to systematically understand the properties and dynamical evolution of various systems with hitherto neglected anomalous effects.

In this paper, we study the collective modes and their consequences in relativistic electromagnetic or quark-gluon plasmas (QGP) at finite temperature T and μ_5 based on the new kinetic theory. (For a review on the collective modes *without* Berry curvature corrections, see Ref. [12].) We show that the Berry curvature corrections dramatically modify the dispersion relation of the collective modes, and in particular, lead to unstable modes, signaling the presence of a plasma instability. We shall call it the chiral plasma instability. Closely related instability was studied (but not by the kinetic theory) within the electroweak theory at large lepton chemical potential [13, 14] and in the context of the early Universe at $T \gg \mu_5$ [15–17] where it is ascribed to a possible origin of the primordial magnetic field.

Our main purpose of this paper is to reveal the poten-

tial importance of the chiral plasma instabilities in heavy ion collisions and compact stars. It has been argued that the QGP created in noncentral heavy ion collisions may contain locally finite μ_5 for quarks and a large external magnetic field to yield an observable CME [18]. It was also suggested that a degenerate electromagnetic plasma with finite μ_5 for electrons may exist inside neutron stars, where the CME is generated in a strong magnetic field [28]. We show within the kinetic theory that these QCD and QED plasmas at finite μ_5 are dynamically unstable and reduce μ_5 by converting it to (color) electromagnetic fields with magnetic helicities. We estimate the typical time scales of the chiral plasma instabilities for the QCD and QED plasmas as $\tau_{\text{QCD}} \sim 1/(\alpha_s^2 \mu_5 \ln \alpha_s^{-1})$ for $\mu_5 \sim T$ and $\tau_{\text{QED}} \sim 1/(\alpha^2 \mu_5)$, respectively.

Kinetic theory with parity-violating effects.—Let us describe a chiral plasma within the kinetic framework. We first consider the collisionless kinetic theory and we shall argue the effect of collisions later. We also consider the regime of a sufficiently weak gauge field A^μ , where there is no essential difference between Abelian and non-Abelian gauge fields. For simplicity of notation, we first consider QED, and we will generalize it to QCD later (with some modifications mentioned below). We assume massless quarks and spatially homogeneous μ_5 below.

Recall the Maxwell equation

$$\partial_\nu F^{\nu\mu} = j_{\text{ind}}^\mu + j_{\text{ext}}^\mu, \quad (1)$$

where j_{ind}^μ is the induced current and j_{ext}^μ is the external current. For the small gauge field A^ν , the induced current can be expressed, via the linear response theory, as

$$j_{\text{ind}}^\mu(K) = \Pi^{\mu\nu}(K) A_\nu(K), \quad (2)$$

in the momentum space, where $\Pi^{\mu\nu}$ is the (retarded) self-energy and $K^\mu = (\omega, \mathbf{k})$ is the four-momentum. From Eqs. (1) and (2), we obtain

$$[K^2 g^{\mu\nu} - K^\mu K^\nu + \Pi^{\mu\nu}(K)]A_\nu(K) = -j_{\text{ext}}^\mu. \quad (3)$$

Due to the gauge invariance, which allows us to shift $A_\nu \rightarrow A_\nu + K_\nu$, we can choose the temporal gauge $A_0 = 0$. Then Eq. (3) can be rewritten, using the physical electric field, as

$$[\Delta^{-1}]^{ij} E^j \equiv [(k^2 - \omega^2)\delta^{ij} - k^i k^j + \Pi^{ij}]E^j = -i\omega j_{\text{ext}}^i, \quad (4)$$

where $k \equiv |\mathbf{k}|$. The dispersion relation for the collective modes in the system can be found by computing the poles of Δ^{ij} ; a mode with the dispersion relation $\omega = \omega(k)$ satisfying $\text{Im}(\omega) > 0$, if exists, implies an instability.

In order to find the explicit form of $\Pi^{\mu\nu}$ including the anomalous parity-violating effects, we use the kinetic theory with Berry curvature corrections [6–8]. Although the same result for $\Pi^{\mu\nu}$ may be obtained in perturbation theory for the isotropic equilibrium distribution function $n_{\mathbf{p}}^0$ [8] (see also Ref. [19]), the kinetic framework would be more versatile; it is easy to include the effect of collisions, as we shall do later. It is also suitable for other future applications, such as the inclusion of the anisotropy of $n_{\mathbf{p}}^0$ and numerical simulations of the dynamical evolution of plasmas.

At the leading order in A^μ , the kinetic theory is given by [8]

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}})n_{\mathbf{p}} + (e\mathbf{E} + \mathbf{v} \times e\mathbf{B} - \nabla_{\mathbf{x}}\epsilon_{\mathbf{p}}) \cdot \nabla_{\mathbf{p}}n_{\mathbf{p}} = 0, \quad (5)$$

where $\mathbf{v} = \mathbf{p}/p$, $\epsilon_{\mathbf{p}} = p(1 - e\mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{p}})$ with $\boldsymbol{\Omega}_{\mathbf{p}} = \pm \mathbf{p}/2p^3$ being the Berry curvature for right- and left-handed fermions, respectively. The energy of the chiral fermion is shifted from $\epsilon_{\mathbf{p}} = p$ by the amount $-pe\mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{p}}$ due to the magnetic moment of chiral fermions at finite μ [8]. This correction makes the term involving $\nabla_{\mathbf{x}}\epsilon_{\mathbf{p}}$ in Eq. (5) nonvanishing for an inhomogeneous magnetic field, unlike the conventional Vlasov equation. In the presence of the Berry curvature flux, the definition of the current is also modified to [6–8]

$$\mathbf{j} = -e^2 \int_{\mathbf{p}} [\epsilon_{\mathbf{p}} \nabla_{\mathbf{p}} n_{\mathbf{p}} + (\boldsymbol{\Omega}_{\mathbf{p}} \cdot \nabla_{\mathbf{p}} n_{\mathbf{p}}) \epsilon_{\mathbf{p}} \mathbf{B} + \epsilon_{\mathbf{p}} \boldsymbol{\Omega}_{\mathbf{p}} \times \nabla_{\mathbf{x}} n_{\mathbf{p}}]. \quad (6)$$

For a moment, we concentrate on right-handed fermions with chemical potential μ . (This theory coupled to dynamical gauge fields itself is not well-defined due to

the gauge anomaly, but we shall eventually consider the theory with both right- and left-handed fermions, so that the gauge anomalies are canceled out.) We solve the linearized equation of (5) in terms of the deviation $\delta n_{\mathbf{p}}$ from the thermal equilibrium state $n_{\mathbf{p}}^0 = 1/[e^{(\epsilon_{\mathbf{p}} - \mu)/T} + 1]$, where $n_{\mathbf{p}} = n_{\mathbf{p}}^0 + \delta n_{\mathbf{p}}$. Note that $n_{\mathbf{p}}^0$ does not obey $n_{\mathbf{p}}^0 = n_{-\mathbf{p}}^0$ and breaks parity.

Substituting the solution $n_{\mathbf{p}}$ to the current \mathbf{j} and using the linear response theory (2), the self-energy of the gauge field can be expressed as $\Pi^{\mu\nu}(K) = \Pi_+^{\mu\nu}(K) + \Pi_-^{\mu\nu}(K)$, where [8]

$$\Pi_+^{\mu\nu}(K) = -m_D^2 \left[\delta^{\mu 0} \delta^{\nu 0} - \omega \int \frac{d\mathbf{v}}{4\pi} \frac{v^\mu v^\nu}{v \cdot K + i\epsilon} \right], \quad (7)$$

$$\Pi_-^{ij}(K) = \frac{e^2 \mu}{4\pi^2} i\epsilon^{ijk} k^k \left(1 - \frac{\omega^2}{k^2} \right) [1 - \omega L(K)], \quad (8)$$

are the parity-even and parity-odd self energies with

$$m_D^2 = e^2 \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right), \quad (9)$$

$$L(K) = \frac{1}{2k} \ln \frac{\omega + k}{\omega - k}. \quad (10)$$

Here i, j, k denote the spatial indices [$\Pi_-^{\mu\nu}(K)$ is vanishing otherwise] and $v = (1, \mathbf{v})$. Note that Π_- is shown to be T -independent from the topological nature of the Berry curvature (see the similar computation of the CME in Ref. [6]).

Collective modes.—We perform a tensor decomposition for the self-energy Π^{ij} . Recall that Π^{ij} is not symmetric with respect to the indices i and j due to the parity-violating term Π_-^{ij} ; we need to use not only the longitudinal and transverse projectors, $P_L^{ij} = \hat{k}^i \hat{k}^j$ and $P_T^{ij} = \delta^{ij} - \hat{k}^i \hat{k}^j$, but also the antisymmetric tensor, $P_A^{ij} = i\epsilon^{ijk} \hat{k}^k$, to fully decompose the tensor structure of Π^{ij} , where $\hat{k}^i = k^i/k$. Then $[\Delta^{-1}]^{ij}$ can be written as

$$[\Delta^{-1}]^{ij} = C_L P_L^{ij} + C_T P_T^{ij} + C_A P_A^{ij}, \quad (11)$$

$$C_L = -\omega^2 + \Pi_L, \quad C_T = -\omega^2 + k^2 + \Pi_T, \quad C_A = \Pi_A,$$

where

$$\Pi_L = -m_D^2 \frac{\omega^2}{k^2} [1 - \omega L(K)], \quad (12a)$$

$$\Pi_T = m_D^2 \frac{\omega^2}{2k^2} \left[1 + \frac{k^2 - \omega^2}{\omega} L(K) \right], \quad (12b)$$

$$\Pi_A = \frac{e^2 \mu k}{4\pi^2} \left(1 - \frac{\omega^2}{k^2} \right) [1 - \omega L(K)]. \quad (12c)$$

In order to compute Δ^{ij} [the inverse of Eq. (11)], it is convenient to use the properties of the (mutually commutable) projectors,

$$P_L^2 = P_L, \quad P_T^2 = P_T, \quad P_L P_T = 0, \quad (13a)$$

$$P_A^2 = P_T, \quad P_L P_A = 0, \quad P_T P_A = P_A, \quad (13b)$$

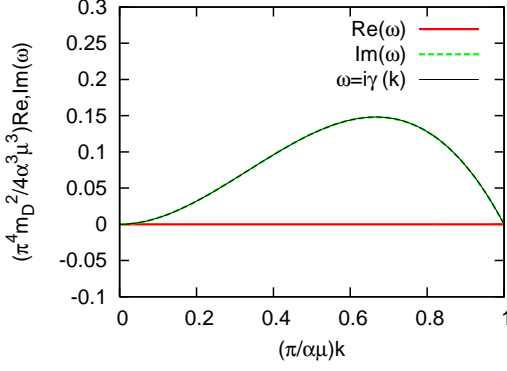


FIG. 1. Real and imaginary parts of the dispersion relation $\omega = \omega(k)$ for the unstable mode.

from which we obtain

$$\Delta^{ij} = \frac{1}{C_L} P_L^{ij} + \frac{C_T}{C_T^2 - C_A^2} P_T^{ij} - \frac{C_A}{C_T^2 - C_A^2} P_A^{ij}. \quad (14)$$

Therefore, the dispersion relations of collective modes are given by $C_L = 0$ and $C_T \pm C_A = 0$, which respectively reduce to

$$\omega^2 = \Pi_L(K), \quad (15)$$

$$\omega^2 = k^2 + \Pi_T(K) \pm \Pi_A(K). \quad (16)$$

While the dispersion relation for the longitudinal component is identical to the case without Π_A , that for the transverse component is split up into two parts by the effect of Π_A .

Let us compute the dispersion relations of the collective modes based on Eqs. (15) and (16).

In the long-wavelength limit $|\omega| \gg k$, Π_A is negligible compared with $\Pi_{L,T}$ in the leading order, and

$$\Pi_{L,T} = \frac{1}{3} m_D^2 \equiv \omega_{\text{pl}}^2. \quad (17)$$

The dispersion relations for the longitudinal and transverse modes are both given by $\omega^2 = \omega_{\text{pl}}^2$, implying that nonstatic gauge fields oscillate with the plasma frequency ω_{pl} .

In the quasi-static limit $|\omega| \ll k$, using $\omega L(K) = \mp \frac{i\pi}{2} x + x^2 + \frac{1}{3} x^4 + \dots$ for positive and negative $\text{Im}(\omega)$ with $x \equiv |\omega|/k \ll 1$, the transverse dispersion relation can be obtained perturbatively in x . Equation (16) with the minus sign has the solutions,

$$\omega = \pm i\gamma(k), \quad \gamma(k) = \frac{4\alpha\mu}{\pi^2 m_D^2} k^2 \left(1 - \frac{\pi k}{\alpha\mu}\right) \quad (18)$$

for $0 \leq k \leq \alpha\mu/\pi$ with $\alpha = e^2/(4\pi)$, while that with the plus sign does not; see Fig. 1 for the numerical result of Eq. (16) in this regime. The mode $\omega = i\gamma(k)$ with

$\gamma > 0$ is unstable and grows exponentially as $e^{-i\omega t} \sim e^{\gamma t}$, indicating the plasma instability. The typical scales of the unstable mode are

$$k \sim \alpha\mu, \quad |\omega| \sim \frac{\alpha\mu k^2}{m_D^2} \sim \alpha^2\mu, \quad (19)$$

and the time scale of the plasma instability is $\tau_{\text{inst}} = 1/\gamma \sim 1/(\alpha^2\mu)$.

Without Π_A originating from Berry curvature corrections, there was no unstable mode for isotropic $n_{\mathbf{p}}^0$. Note also that the criteria for the conventional (Weibel) plasma instability in Ref. [20] is not applicable here. This is because the assumptions there, $n_{\mathbf{p}}^0 = n_{-\mathbf{p}}^0$ and $\Pi^{ij} = \Pi^{ji}$, are violated by the parity-violating effects.

Now these results can easily be extended to a plasma at finite vector and chiral chemical potentials, $\mu_V = \mu_R + \mu_L$ and $\mu_5 = \mu_R - \mu_L$, by the following replacements: $\mu^2 \rightarrow \mu_R^2 + \mu_L^2 = \frac{1}{2}(\mu_V^2 + \mu_5^2)$ in m_D^2 , and $\mu \rightarrow \mu_R - \mu_L = \mu_5$ in Π_{-} , Π_A and Eq. (18).

The fate of chiral plasma instabilities.—What is the fate of chiral plasma instabilities above? Below we shall argue that this instability reduces μ_5 , so that it is weakened. We first recall the relation of triangle anomalies,

$$\partial_t Q_5 = \frac{2\alpha}{\pi} \int_{\mathbf{x}} \mathbf{E} \cdot \mathbf{B}, \quad (20)$$

where Q_5 is the global chiral charge. Equivalently,

$$\partial_t \left(Q_5 + \frac{\alpha}{\pi} \mathcal{H} \right) = 0, \quad \mathcal{H} = \int_{\mathbf{x}} \mathbf{A} \cdot \mathbf{B}, \quad (21)$$

where \mathcal{H} is the Chern-Simon number (which is also called the magnetic helicity in plasma physics). Equation (21) is the conservation of the helicity.

We now explicitly show the reduction of μ_5 within the kinetic theory in the regime under consideration. We concentrate on the unstable mode, $C_T \simeq C_A$ at $|\omega| \ll k$, for which the longitudinal part is negligible. Using Eq. (14), $A^i = \Delta^{ij} j_{\text{ext}}^j$ [see Eq. (4)] reduces to

$$A^i(K) \simeq \frac{i\beta(k)}{\omega - i\gamma(k)} J_{\text{ext}}^i(K), \quad (22)$$

where we defined $\beta(k) = -2k/(\pi m_D^2)$ and $J_{\text{ext}}^i(K) \equiv (P_T^{ij} - P_A^{ij}) j_{\text{ext}}^j(K)$. In the (t, \mathbf{k}) space, A^i satisfies

$$[\partial_t - \gamma(k)] A^i(t, \mathbf{k}) = \beta(k) J_{\text{ext}}^i(t, \mathbf{k}). \quad (23)$$

This can be solved in terms of A^i for $j_{\text{ext}}(t, \mathbf{k}) = j_{\text{ext}}(\mathbf{k})\delta(t)$:

$$A^i(t, \mathbf{k}) = \beta e^{\gamma t} \theta(t) J_{\text{ext}}^i(\mathbf{k}). \quad (24)$$

We then obtain $E^i = -\partial_t A^i$ and $B^i = -k P_A^{ij} A^j$ as

$$E^i(t, \mathbf{k}) = -\beta[\gamma e^{\gamma t} \theta(t) + \delta(t)] J_{\text{ext}}^i(\mathbf{k}), \quad (25)$$

$$B^i(t, \mathbf{k}) = k\beta e^{\gamma t} \theta(t) J_{\text{ext}}^i(\mathbf{k}), \quad (26)$$

where we used $P^A J_{\text{ext}} = -J_{\text{ext}}$ that follows from Eq. (13b). Hence we find $\mathbf{E}(t, \mathbf{k}) \cdot \mathbf{B}^*(t, \mathbf{k}) < 0$ and $\mathbf{A}(t, \mathbf{k}) \cdot \mathbf{B}^*(t, \mathbf{k}) > 0$. From Eq. (20), this unstable mode decreases μ_5 for fixed T and increases the magnitude of \mathcal{H} . As a result, it weakens the instability.

The concrete time evolution of the plasma beyond the leading order in A^μ can be described by the full kinetic theory [see Eq. (15) in Ref. [8]] together with Eqs. (1), (6) and (20), which would require a numerical analysis. Still the saturation of the instability itself may be understood from the energy and helicity conservations. We assume $\mu_5 \sim T$. As the energy and magnetic helicity of electromagnetic fields, $\sim (kA)^2$ and $\sim \alpha k A^2$, come from those of chiral fermions, $O(\mu_5^4)$ and $O(\mu_5^3)$, we get the typical scales of k and A relevant to the instability:

$$k \sim \alpha \mu_5, \quad A \sim \frac{\mu_5}{\alpha}, \quad B \sim \mu_5^2. \quad (27)$$

The electromagnetic fields cannot grow beyond this order and will saturate in the end. At the saturation, the gauge fields become nonperturbatively large.

Effect of collisions.—For $\mu_5 \sim T$, the mean free time between scattering of the plasma constituents in QED, $\tau_{\text{mft}} \sim 1/(\alpha^2 T \ln \alpha^{-1})$ [21], is shorter than the time scale of the plasma instability, $\tau_{\text{inst}} \sim 1/(\alpha^2 \mu_5)$. On the other hand, the mean free path, $l_{\text{mfp}} \sim \tau_{\text{mft}}$, is larger than the typical wavelength of the plasma instability, $1/k_{\text{inst}} \sim 1/(\alpha \mu_5)$, and one expects that the effect of collisions is irrelevant for it.

To see this explicitly, consider the effect of collisions in the relaxation time approximation: we add the term $-\delta n_{\mathbf{p}}/\tau_{\text{rel}}$ in the right-hand side of Eq. (5), where τ_{rel} is the relaxation time which is assumed to be of the same order as τ_{mft} . Then ω in the denominator of Eq. (7) is replaced by $\omega + i/\tau_{\text{rel}}$. Repeating a similar computation to above, one finds that the modification to Eq. (18) by the effect of collisions is subleading in $1/(k_{\text{inst}} \tau_{\text{rel}}) \ll 1$ and is negligible. (From this argument, the effect of collisions is clearly negligible for $\mu_5 \gg T$.) As we shall see below, however, this is not the case in QCD.

Quark-gluon plasma with μ_5 .—Suppose μ_5 ($\sim T$) is initially generated in the QGP before the thermalization, whose evolution is described by the kinetic theory. The above arguments are then extended to color electromagnetic fields, within the leading order in A , by replacing $\alpha \rightarrow \alpha_s = g^2/(4\pi)$ and $m_D^2 \rightarrow (N_f + 2N_c)g^2 T^2/6 + N_f g^2 \mu^2/(2\pi^2)$ with N_f and N_c being the number of

flavors and colors. The qualitative difference from the QED plasma is that the mean free time (path) is now $\tau_{\text{mft}} \sim l_{\text{mfp}} \sim 1/(\alpha_s T \ln \alpha_s^{-1})$ due to the fact that colored gluons, exchanged between quarks, can change the color charges of the scatterers [22–25]. As $\tau_{\text{mft}} \ll \tau_{\text{inst}}$ and $l_{\text{mfp}} \ll 1/k_{\text{inst}}$ in this case, the collisions can affect the plasma instability. In the relaxation time approximation, $L(\omega + i/\tau_{\text{rel}}, k) \simeq -i\tau_{\text{rel}}$, and one indeed finds that Eq. (18) is modified to

$$\gamma(k) = \frac{2N_f \alpha_s \mu_5 k}{\pi m_D^2 \tau_{\text{rel}}} \left(1 - \frac{\pi k}{\alpha_s \mu_5} \right). \quad (28)$$

The unstable modes in $0 \leq k \leq \alpha_s \mu_5/\pi$ still exist but grow more rapidly with $\tau_{\text{inst}} \sim 1/(\alpha_s^2 \mu_5 \ln \alpha_s^{-1})$.

Conclusion and outlook.—We have shown that the relativistic electromagnetic plasma and QGP with finite μ_5 are dynamically unstable and μ_5 is damped by the exponential growth of (color) electromagnetic fields at the early stage [26]. As the gauge field grows, the nonlinearity of the gauge field becomes important; the detailed analysis for the QGP would require the kinetic theory with non-Abelian Berry curvatures, which is beyond the scope of the present paper.

To understand the roles of the chiral plasma instabilities at the later stage more quantitatively, detailed studies of the dynamical evolution of plasmas based on the kinetic theory need to be worked out. There are also other directions in which one can extend or improve our analysis: (i) It is straightforward to include the anisotropy of $n_{\mathbf{p}}^0$ analogous to Ref. [29]. This allows for the study of the coexistence (or competition) with the Weibel instability. (ii) The effect of the quark mass can be incorporated in the chirality evolution in Eq. (20), which makes μ_5 smaller. These analyses would be important to understand whether the CME remains an observable effect in the realistic situations.

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